# Interactions between one-dimensional quadratic solitons

**Y. Baek, R. Schiek, and G. I. Stegeman**

*Center for Research and Education in Optics and Lasers, University of Central Florida, 4000 Central Florida Boulevard, Orlando, Florida 32816-2700*

**I. Baumann and W. Sohler**

*Angewandte Physik, Universität Paderborn, Warburger Strasse 100, D-33098 Paderborn, Germany*

Received July 7, 1997

The interaction between two one-dimensional quadratic solitons has been investigated experimentally in lithium niobate planar waveguides for both parallel- and crossing-launched solitons. 

A variety of optical bright spatial solitons have been investigated experimentally in the past few years.[1] In addition to the usual Kerr solitons, Manakov, photorefractive, and quadratic solitons have been observed.[2–4] Of these, the quadratic solitons are unique because they do not utilize refractive-index changes for self-trapping and they consist of a minimum of two strongly coupled waves of different frequencies linked by means of a second-order nonlinear interaction. Furthermore, the detuning from the phase-matching condition determines both the shape of the soliton beams and the admixture of the interacting fields. The existence of quadratic solitons was predicted in 1975, and many of their properties have been explored theoretically in recent years.[5–7] The quadratic solitons associated with second-harmonic (SH) generation have been observed in geometries in which a beam can spatially diffract in one (in slab waveguides) and two (in bulk media) dimensions.[4,8] Because of their multibeam and multifrequency nature, the interactions between quadratic solitons are expected to exhibit features different from those observed previously for Kerr solitons.[9–13] In this Letter we report experimental investigations of the interaction between one-dimensional quadratic solitons in planar $LiNbO_3$ waveguides.

We previously reported the generation of one-dimensional quadratic solitons along the $x$ axes of $y$-cut, planar, Ti:indiffused $LiNbO_3$ waveguides near the phase-matching conditions for type I SH generation at 1320 nm.[4] The depth dependence of the refractive index provided the guided mode confinement along the $y$ axis. Mutual beam trapping occurs in the plane of the waveguide, i.e., along the $z$ axis. The same samples were used in the research reported here. They require heating inside an oven to ~335 °C to achieve phase matching. The resulting sample temperature distribution is uniform in the center of the waveguide but drops a few degrees near the oven windows; i.e., the resulting wave-vector mismatch varies with distance along the waveguide.[4] Although this result complicated the analysis of the quadratic solitons, the solitons were still easily excited at a positive phase mismatch of $\sim 10\pi$ at $T = 335.05$ °C at the center of the oven. Because the SH component of the solitons is small (<10%) relative to the fundamental for a detuning this large, the quadratic solitons resemble Kerr solitons based on $\chi^{(3)}$. They can be excited with just a fundamental input and evolve into stationary solitons as the required SH is generated with distance into the sample. In this research the solitons were generated both far from phase matching (as before) and close to phase matching ($3\pi$, $T = 335.35$ °C) when the soliton contained ~50% SH.

A $Q$-switched mode-locked Nd:YAG laser with 90-ps pulses was used to generate the solitons at 1320 nm. A combination of optical elements was used to generate two separate, equipower beams, one of which was delayed relative to the other to produce a well-defined relative phase difference at the sample input. Cylindrical lenses focused elliptically shaped beams onto the sample input facet. The output from the 47-mm-long planar sample (approximately three diffraction lengths long) was focused onto a vidicon camera for display.

Two types of interaction were investigated. First, two $y$-polarized fundamental beams, each 70 $\mu$m FWHM, were launched parallel to each other at a center-to-center separation of 138 $\mu$m. The experimental results in Fig. 1 show soliton repulsion for a phase difference $\varphi = \pi$ and that there is a power exchange for $\varphi = \pi/2, 3\pi/2$. The power was transferred from the beam that leads in phase to the one that lags in phase, as for the Kerr case.[13] The two beams seemed to fuse when they were in phase.

The interaction was modeled with coupled-mode theory for Fourier spectra. Parallel to the film, the $z$ dependence of the fields $E_i(y,z)$ at every position $x$ along the waveguide could be expanded as spatial Fourier integrals in the form $E_i(y,z) = [e_i(y)/2\pi] \int \mathrm{d}\beta_z A_i(\beta_z) \exp(i\beta_z)$. The upconversion and downconversion processes are described by

$$\frac{\mathrm{d}}{\mathrm{d}x} A_1(\beta_z) + i\beta_{1x} A_1(\beta_z) = -i\,\frac{\omega_1 K^{(2)} 2\chi^{(2)} \beta_1}{8\pi p_0 \beta_{1x}} \times \int \mathrm{d}\beta_z{}' A_2(\beta_z - \beta_z{}') A_1{}^*(-\beta_z{}'), \quad (1)$$

$$\frac{\mathrm{d}}{\mathrm{d}x} A_2(\beta_z) + i\beta_{2x} A_2(\beta_z) = -i\,\frac{\omega_1 K^{(2)} 2\chi^{(2)} \beta_2}{8\pi p_0 \beta_{2x}} \times \int \mathrm{d}\beta_z{}' A_1(\beta_z - \beta_z{}') A_1(\beta_z{}'). \quad (2)$$

0146-9592/97/201550-03$10.00/0 

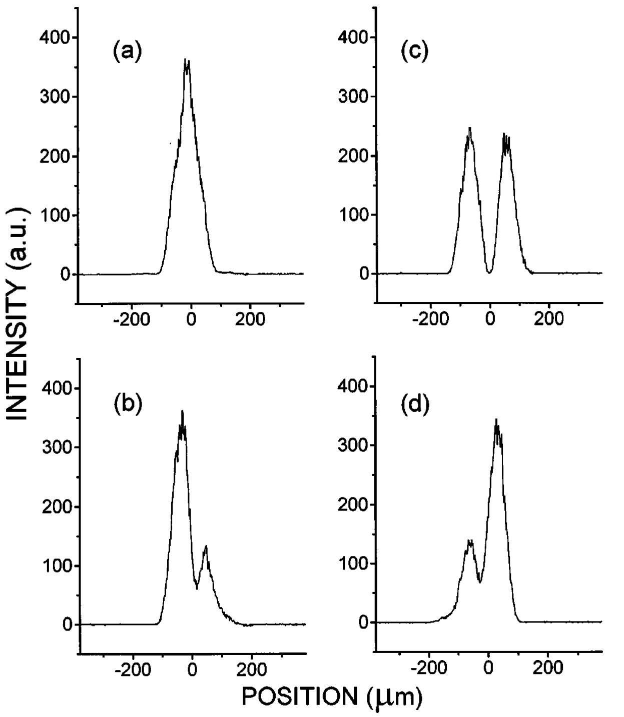


Fig. 1. Measured output beam profiles of the two-solitary-wave interaction for the parallel-launching case for large phase mismatch ($T = 335.05\,°\mathrm{C}$). The relative phase difference between the two beams is (a) 0, (b) $\pi/2$, (c) $\pi$, (d) $3\pi/2$.

Here the modal field distributions are given by $e_i(y)$ and $i = 1, 2$ identify the fundamental and the harmonic beams. $\beta_z$ is the spatial angular frequency, $\beta_{ix} = ({\beta_i}^2 - {\beta_{iz}}^2)^{1/2}$ are the $x$ components of the mode propagation constants $\beta_i$, and $p_0$ is the normalized mode power per unit film width (in watts per meter). $K^{(2)} = \int dy {e_1}^2(y) {e_2}^*(y)$ is the overlap integral, which takes into account the different transverse electric field profiles of the interacting guided modes.

The results of numerical simulations based on the experimental parameters as input are shown in Fig. 2, and the agreement with experiment is excellent. However, when simulations for uniform wave-vector mismatch with distance are allowed to proceed to longer propagation distances than are accessible experimentally, the in-phase interaction actually leads to a periodic collapse and recovery to progressively smaller soliton transverse separations and eventually to fusion of the two solitons. In this experiment the sample length was just long enough that we could see the first collapse. In summary, for this far-off phase-matching case the soliton interactions resemble those of Kerr solitons for the sample lengths used.[9,13] Longer sample lengths are needed to demonstrate the behavior that is characteristic of saturable nonlinear media.

Nearer phase matching, the situation is more complicated. Qualitatively the same behavior was observed, but at an earlier stage of the interaction because now 45% of the SH was needed to yield a stationary soliton and it took a longer propagation distance for the input beam to evolve into a solitary wave. The results, shown in Figs. 3 and 4, illustrate the incomplete nature of the interactions. Given the launching condition uncertainties, the simulations were in excellent agreement with the experiment. We note, however, that for

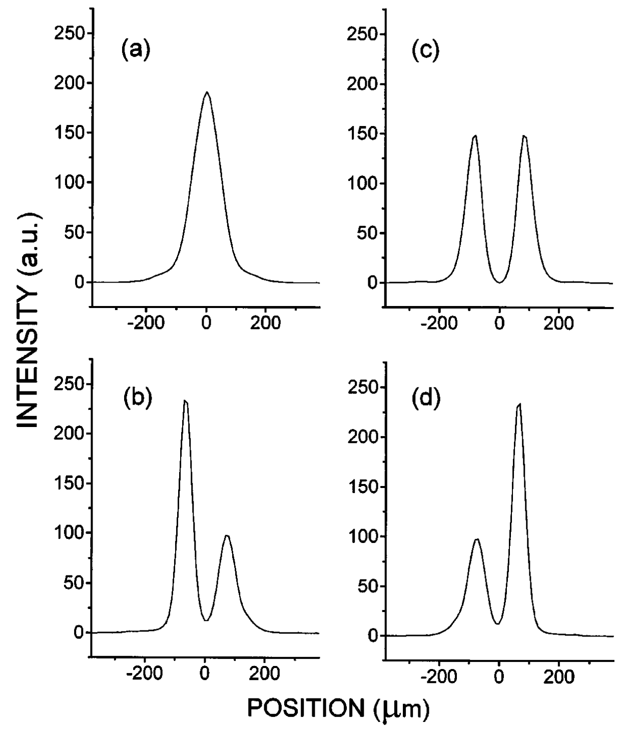


Fig. 2. Numerical simulations of the two-solitary-wave interaction for the parallel-launching case for large phase mismatch ($T = 335.05\,°\mathrm{C}$). The relative phase difference between the two beams is (a) 0, (b) $\pi/2$, (c) $\pi$, (d) $3\pi/2$.

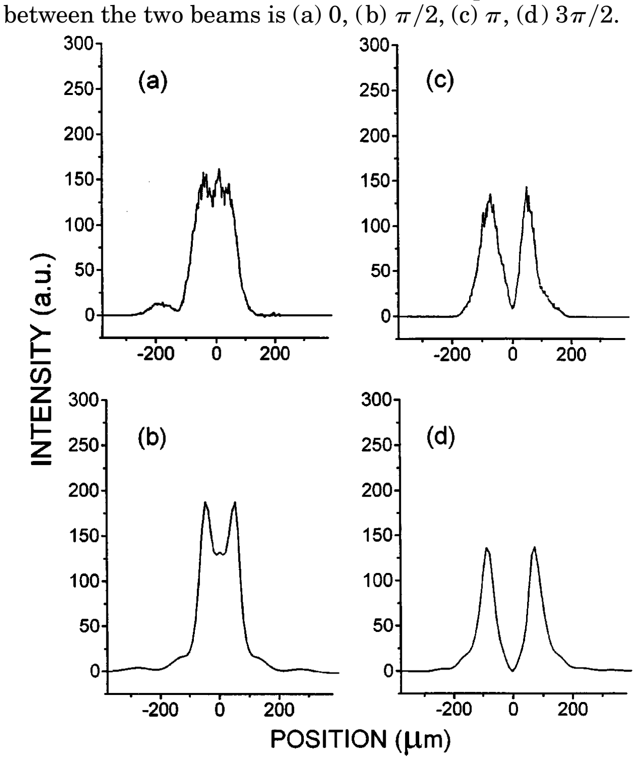


Fig. 3. (a), (c) Measured and (b), (d) simulated output beam profiles of the two-solitary-wave interaction for the parallel-launching case for small phase mismatch ($T = 335.35\,°\mathrm{C}$). The relative phase difference between the two beams is (a), (b) 0 and (c), (d) $\pi$.

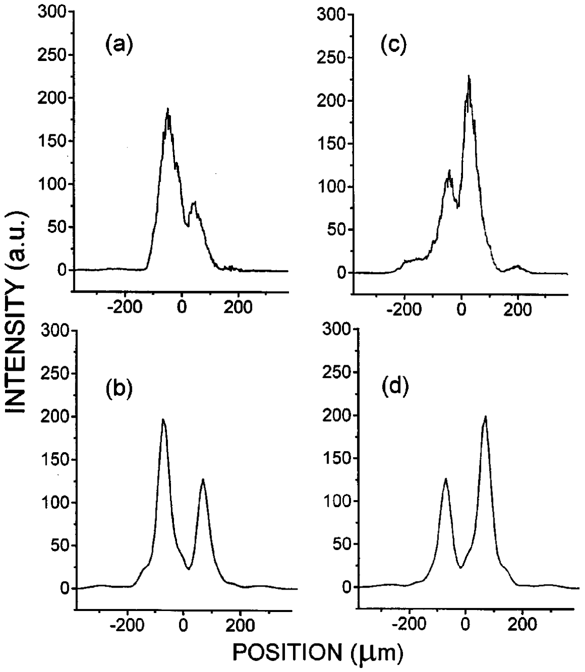


Fig. 4. (a), (c) Measured and (b), (d) simulated output beam profiles of the two-solitary-wave interaction for the parallel-launching case for small phase mismatch ($T$ = 335.35 °C). The relative phase difference between the beams is (a), (b) $\pi/2$ and (c), (d) $3\pi/2$.

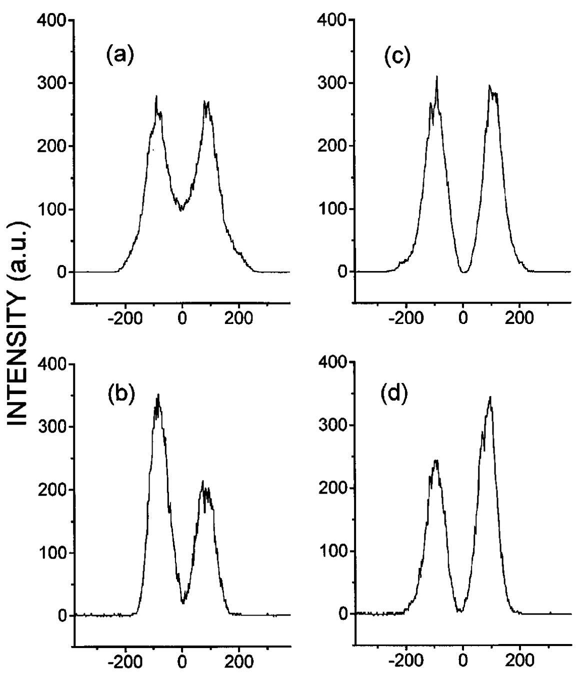


Fig. 5. Measured output beam profiles for the two-solitary-wave interaction in the cross-launching case for large net phase mismatch ($T$ = 335.05 °C). The relative phase difference between the two beams is (a) 0, (b) $\pi/2$, (c) $\pi$, (d) $3\pi/2$.

the in-phase case detailed numerical simulations show that the two beams fuse and no periodic behavior occurs, reminiscent of saturable Kerr media.[14]

In the second geometry we launched the two fundamental beams, initially separated by 98 $\mu$m, at a collision angle inside the crystal of 0.42° to investigate the collision of two crossing solitons. Figure 5 shows the output beam profiles at large phase mismatch ($T$ = 335.05 °C). For $\varphi = 0$, $\pi$, the two beams passed through each other. For $\varphi = \pi/2$, $3\pi/2$, a significant amount of energy transfer was observed. Again the results were in agreement with the numerical simulations. Because the collision input caused the soliton fields to overlap strongly in the middle of the sample, their interaction behavior deviated more obviously from that associated with Kerr solitons than for the parallel-launching case.

In conclusion, the interaction between quadratic solitary waves was studied both numerically and experimentally. Near phase matching, the unique features of quadratic solitary-wave interactions were readily observed and resembled interactions in saturable Kerr media, which was a consequence of the strong SH generation involved in the interaction. Far from phase matching, the interaction behavior resembled more closely that of Kerr solitons. Additionally, stronger effects have been observed in cross- than in parallel-launching geometries.

This research was supported by the National Science Foundation and the U.S. Air Force Office of Scientific Research.

### References


1. G. I. Stegeman, Opt. Appl. **26,** 239 (1997).
2. J. U. Kang, G. I. Stegeman, J. S. Aitchison, and N. Akhmediev, Phys. Rev. Lett. **76,** 3699 (1996).
3. G. C. Duree, Jr., J. L. Shultz, G. J. Salamo, M. Segev, A. Yariv, B. Crosignani, P. Di Porto, E. J. Sharp, and R. R. Neurgaonkar, Phys. Rev. Lett. **71,** 533 (1993).
4. R. Schiek, Y. Baek, and G. I. Stegeman, Phys. Rev. E **53,** 1138 (1996).
5. Yu. N. Karamzin and A. P. Sukhorukov, Zh. Eksp. Teor. Fiz. **68,** 834 (1975) [Sov. Phys. JETP **41,** 414 (1976)].
6. D. E. Pelinovsky, A. V. Buryak, and Y. S. Kivshar, Phys. Rev. Lett. **75,** 591 (1995).
7. L. Torner, C. R. Menyuk, W. E. Torruellas, and G. I. Stegeman, Opt. Lett. **20,** 13 (1995).
8. W. E. Torruellas, Z. Wang, D. J. Hagan, E. W. Van Stryland, G. I. Stegeman, L. Torner, and C. R. Menyuk, Phys. Rev. Lett. **74,** 5036 (1995).
9. J. S. Aitchison, A. M. Weiner, Y. Silberberg, D. E. Leaird, M. K. Oliver, J. L. Jackel, and P. W. E. Smith, Opt. Lett. **16,** 15 (1991).
10. A. V. Buryak, Y. S. Kivshar, and V. V. Stelbina, Phys. Rev. A **52,** 1670 (1995).
11. D.-M. Baboiu, G. I. Stegeman, and L. Torner, Opt. Lett. **20,** 2282 (1995).
12. C. Etrich, U. Peschel, F. Lederer, and B. Malomed, Phys. Rev. A **52,** 3444 (1995).
13. M. Shalaby, F. Reynaud, and A. Barthelemy, Opt. Lett. **17,** 778 (1992).
14. A. W. Snyder and A. P. Sheppard, Opt. Lett. **18,** 482 (1993).